Enhancing cognitive flexibility through a training based on multiple categorization: developing proportional reasoning in primary school

Améliorer la flexibilité cognitive par une intervention fondée sur la catégorisation multiple : développer le raisonnement proportionnel à l'école primaire

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Gvozdic, K., & Sander, E. (2020). Learning to be an opportunistic word problem solver: Going beyond informal solving strategies. *ZDM Mathematics Education, 52(1)*, 111-123.

Journal of Numerical Cognition

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Empirical Research

Enhancing Cognitive Flexibility Through a Training Based on Multiple Categorization: Developing Proportional Reasoning in Primary School

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Abstract

Proportional reasoning is a key topic both at school and in everyday life. However, students are often misled by their preconceptions regarding proportions. Our hypothesis is that these limitations can be mitigated by working on alternative ways of categorizing situations that enable more adequate inferences. Multiple categorization triggers flexibility, which enables reinterpreting a problem statement and adopting a more relevant point of view. The present study aims to show the improvements in proportional reasoning after an intervention focusing on such a multiple categorization. Twenty-eight 4th and 5th grade classes participated in the study during one school year. Schools were classified by the SES of their neighborhood. The experimental group received 12 math lessons focusing on flexibly envisioning a situation involving proportional reasoning from different points of view. At the end of the school year, compared to a control group, the experimental group had better results on the posttest when solving proportion word problems and proposed more diverse solving strategies. The analyses also show that the performance gap linked to the school's SES classification was reduced. This offers promising perspectives regarding multiple categorization as a path to overtake preconceptions and develop cognitive flexibility at school.

Keywords

mathematical flexibility, multiple categorization, proportional reasoning, evidence-based education, preconceptions

ZDM
https://doi.org/10.1007/s11858-019-01114-z

ORIGINAL ARTICLE

Learning to be an opportunistic word problem solver: going beyond informal solving strategies

Katarina Gvozdic[1] · Emmanuel Sander[1]

Accepted: 4 December 2019

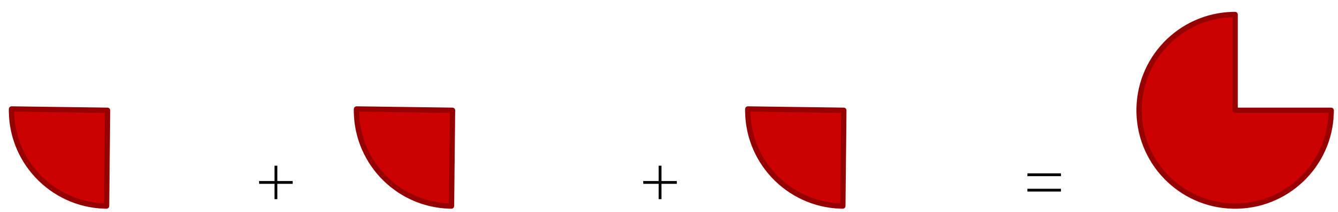
Abstract

Informal strategies reflecting the representation of a situation described in an arithmetic word problem mediate students' solving processes. When the informal strategies are inefficient, teaching students to make way for more efficient ways to find the solution is an important educational issue in mathematics. The current paper presents a pedagogical design for arithmetic word problem solving, which is part of a first-grade arithmetic intervention (ACE). The curriculum was designed to promote adaptive expertise among students through semantic analysis and recoding, which would lead students to favor the more adequate solving strategy when several options are available. We describe the ways in which students were taught to engage in a semantic analysis of the problem, and the representational tools used to favor this conceptual change. Within the word problem solving curriculum, informal and formal solving strategies corresponding to the different formats of the same arithmetic operation, were comparatively studied. The performance and strategies used by students were assessed, revealing a greater use of formal arithmetic strategies among ACE classes. Our findings illustrate a promising path for going past informal strategies on arithmetic word problem solving.

Keywords Arithmetic word problem solving · Informal strategies · Arithmetic knowledge · Mathematics education · Adaptive expertise · Semantic recoding

Finding quarters

Is 3x ¼ = ¾ ? It looks pretty easy, isn't it ?

Is 

?

There are 3 pizzas and 4 kids. Each kid takes their own part of the pizza. 65% vs. 5% of success among 4th graders depending on the initial quantity : if students have to share among 3 pizzas or 1 pizza (Brissiaud, 2003)

Finding quarters

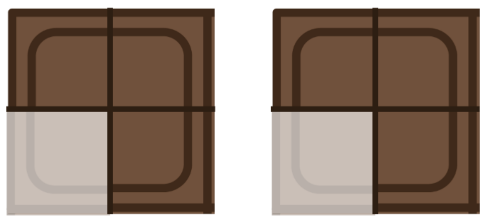
J'ai mangé **un quart de 2 carreaux** de chocolats.

Propose plusieurs façons, écris en Math et en Français

Math Course

(4th & 5h grade)

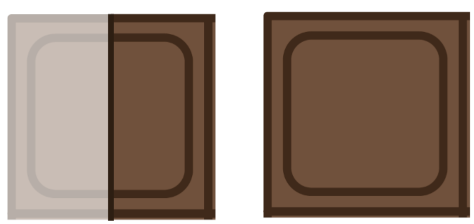
Point of view: Parts



En math : $\frac{1}{4} + \frac{1}{4} = \frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$

En français : *Un quart de chaque carreau*

Point of view: Whole



En math : $\frac{2}{4} \times 1 = \frac{2}{4} = \frac{1}{2}$

En français : *Deux quarts de un carreau*

Finding quarters

Combien y a-t-il de quarts d'heure dans $\frac{3}{4}$ d'heure ?

Cocher la bonne réponse :

	6e	4e
○ 3	50,4%	61,9%
○ 4	13,4%	11,2%
○ 15	14,8%	10,4%
○ 45	16,3%	14,%
No answer	5,1%	2,4%

660 6th grade students
741 8th grade students

DEPP, 2022

Enhancing cognitive flexibility through a training based on multiple categorization: developing proportional reasoning in primary school

Students' informal solving strategies are often seen as the basis for developing formal mathematics knowledge.

In certain contexts, informal knowledge leads to an encoding of a problem favorable for finding the solution, but at other times, it can lead to costly solving strategies and inaccurate answers.

We investigated the benefits of semantic recoding pedagogical interventions (ACE and RAIFLEX) designed to help students overcome their intuitive conceptions and enhance cognitive flexibility in order to use the most appropriate solving strategy – i.e. promote adaptive expertise.

The key phase in both interventions provided students with the means to put aside the initial encoding of the situation and create a new representation, thus making it possible to use a more efficient solving strategy and more in line with the mathematical knowledge which is targeted.

What are informal strategies?

Informal strategies are strategies that do not require the use of mathematical knowledge that is supposedly at play in the situation.

From a pack of 200 images, we make piles of 50 images. How many piles are there? (72% correct)

We divide 200 images into 50 piles. How many images are there in each pile? (21% correct)

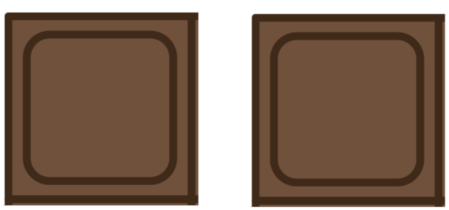
Brissiaud, R., & Sander, E. (2010). Arithmetic word problem solving: a Situation Strategy First Framework. *Developmental Science, 13(1)*, 92-107.

What does encoding mean?

The properties perceived as structuring and relevant from the solver's point of view and according to which they will structure their representation of the mathematical situation.

For a mosquito a human is encoded as « potential food ».

For students, encoding mathematical properties (e.g. a part-whole structure) rather than daily life properties (e.g. a « buying food at a grocery store » problem) is a true challenge that requires education to being overcome.



Do you see one whole or two wholes ?
A person who encodes two wholes will take ¼ of each piece of chocolate
A person who encodes one whole, whe will take ½ of one of the pieces

What does encoding mean?

Mrs. Martin buys in a bookstore for each of her 5 children, 3 pens. How many pens does she buy in total?

You have to perform an addition

3+3+3+3+3

What does encoding mean?

Mrs. Martin buys in a bookstore for each of her 5 children, 3 pens:

1 red pen, 1 bleu pen and one yellow pen.

How many pens does she buy in total?

You have to perform an addition

3+3+3+3+3

BUT also 5+5+5

What does encoding mean?

Mrs. Martin buys in a bookstore for each of her 5 children, 3 pens:
1 red pen, 1bleu pen and one yellow pen.
How many pens does she buy in total?
You have to perform an addition

3+3+3+3+3

BUT also 5+5+5

Two alternative encodings of a same situation, one focusing on the number of pens by child, and the other on the number of children corresponding to each color of pen.

What does encoding mean?

Mrs. Durand buys in a bookstore for each of her 5 children, 3 red pens, 6 blue pens and 4 green pens. How many pens does she buy in total?

5x3 + 5x6 + 5x4

BUT ALSO 5x(3+6+4)

Two alternative encodings of a same situation that makes understandable the distributive property.

Enhancing cognitive flexibility through a training based on multiple categorization: developing proportional reasoning in primary school

Maria has 39 marbles.
During recess, she gains some.
Now she has 42 marbles.
How many did she gain?

Luc has 42 marbles.
During recess, he loses 39 marbles.
How many marbles does Luc have now?

These two problems are analogous (they are both subtraction problems), but this is a difficult pedagogical challenge to help students recognizing the analogy between them, because they are encoded differently.

Which problem is more difficult for 2nd grade students to solve?

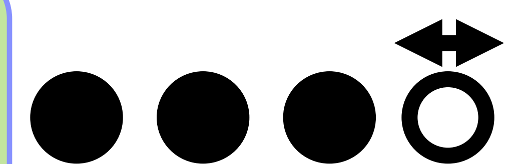
Maria has 39 marbles.
During recess, she gains some.
Now she has 42 marbles.
How many did she gain?

Luc has 42 marbles.
During recess, he loses 39 marbles.
How many marbles does Luc have now?

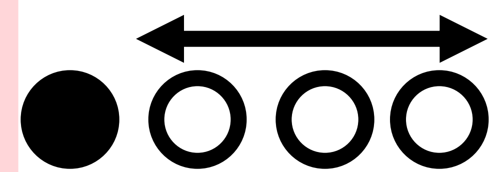
The green one. Why ?

Informal strategies in the driving seat of arithmetic problem solving

Maria has 39 marbles.
During recess, she gains some.
Now she has 42 marbles.
How many did she gain?



Luc has 42 marbles.
During recess, he loses 39 marbles.
How many marbles does Luc have now?

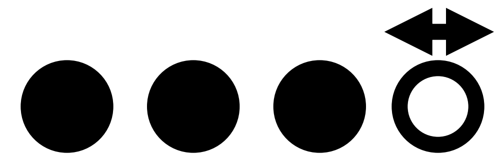


Despite them both being subtraction problems involving the same numerical values, the ’’green’’ version is much easier than the “pink” one (49% vs. 27%)

Informal strategies in the driving seat of arithmetic problem solving

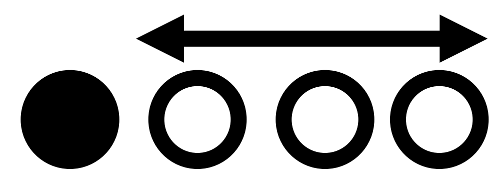
Mental simulation
= Low cost

There are 31 oranges and 27 pears in the basket. How many pears are there less than oranges in the basket?



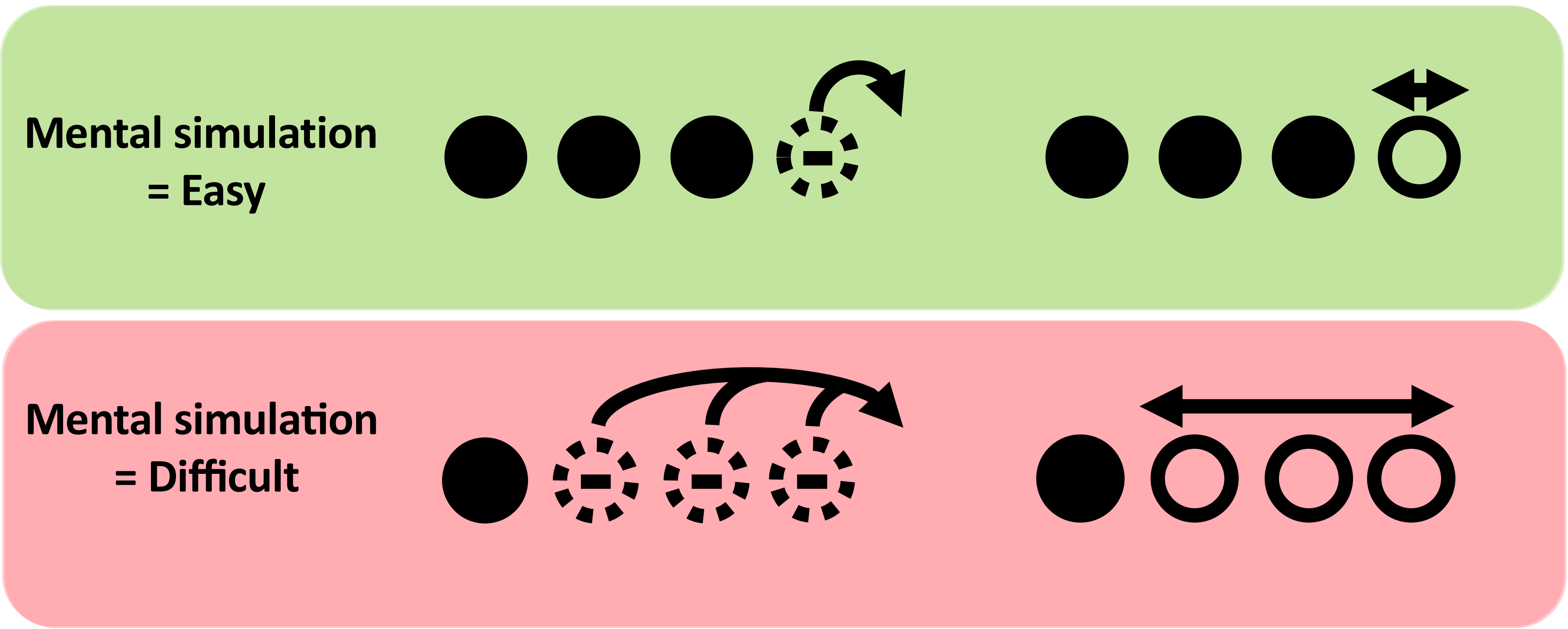
Mental simulation
= High cost

There are 31 oranges and 4 pears in the basket. How many pears are there less than oranges in the basket?



Gvozdic, K., & Sander, E. (2017). Solving additive word problems: Intuitive strategies make the difference. In B. C. Love, K. McRae, & V. M. Sloutsky (Eds.), *Proceedings of the 39th Annual Conference of the Cognitive Science Society* (pp. 2150-2155). London, UK: Cognitive Science Society.

Informal strategies in the driving seat of arithmetic problem solving



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The SSF Model

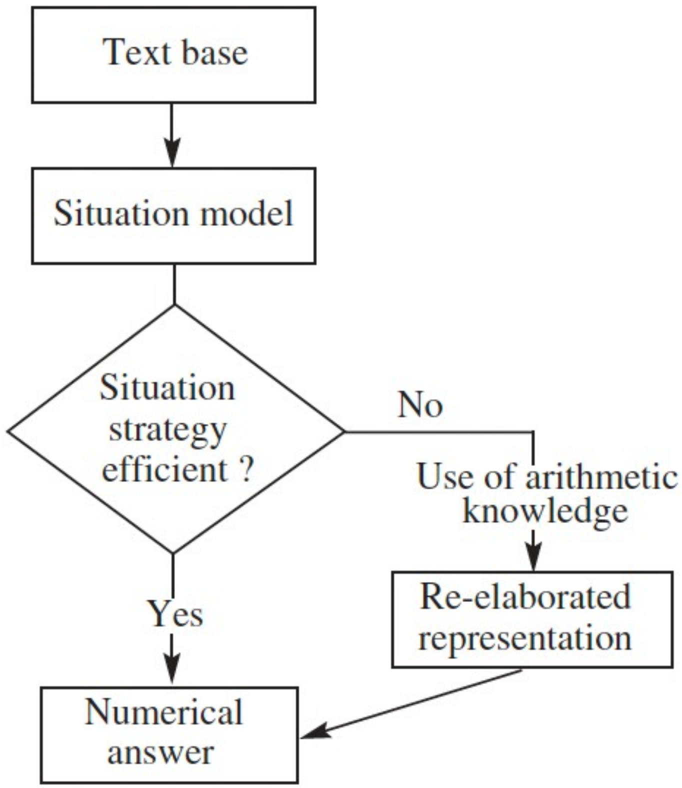


Figure 1 *Architecture of a Situation Strategy First framework.*

Brissiaud, R., & Sander, E. (2010). Arithmetic word problem solving: a Situation Strategy First Framework. *Developmental Science, 13(1)*, 92-107.

The SSF Model

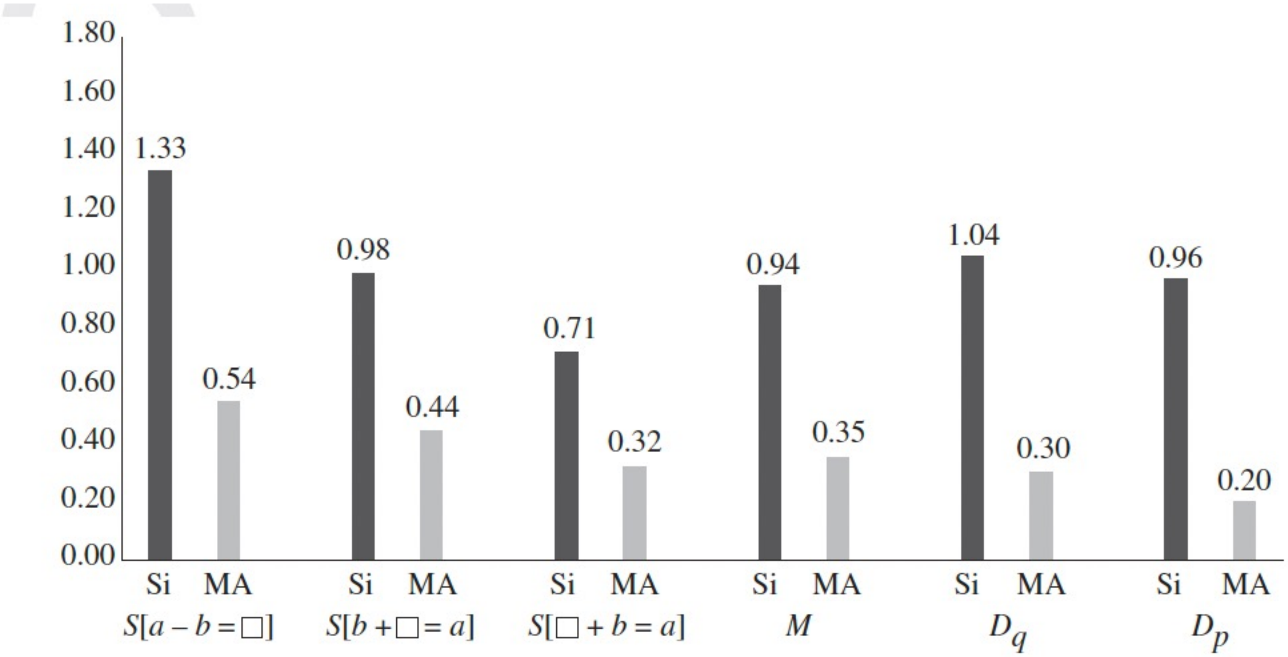


Figure 2 *Average scores for the problems at the beginning of Year 3 (Experiment 1).*

Brissiaud, R., & Sander, E. (2010). Arithmetic word problem solving: a Situation Strategy First Framework. *Developmental Science, 13(1)*, 92-107.

The SSF Model

Why is it much easier to find the number of pens in two packs of 10 pens than in 10 packs of 2 pens?

What are intuitive conceptions?

A notion is perceived by analogy with a familiar knowledge, acquired through daily life.

Intuitive conceptions determine initial representations

They are self-imposed, implicit and robust

Invent a subtraction problem whose solution is 8-3=5

Paul (Hugo, Theo, Nathan, Lea, Marie, Judith, etc.) has 8 candies (marbles, cakes, apples, etc.). He/She gives (eats, loses, etc.) 3 to (during, etc.). How many does he/she have left?

To subtract is to lose, to withdraw, to take away. A totality is given, of which a part is subtracted. The question is about the remaining part.

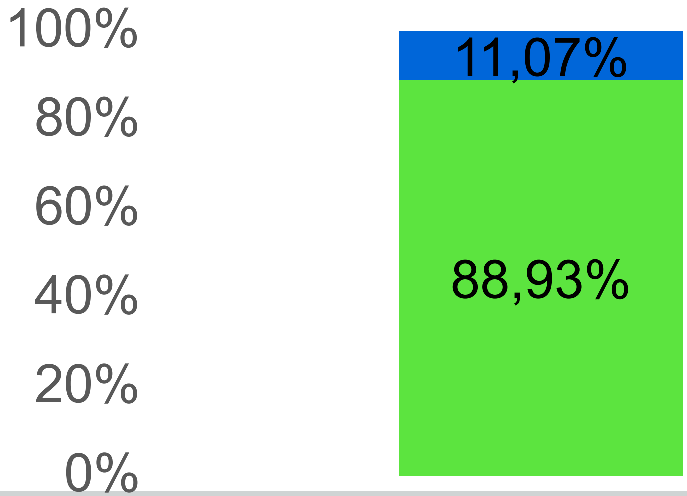
What are intuitive conceptions?

A notion is perceived by analogy with a familiar knowledge, acquired through daily life.

Intuitive conceptions determine initial representations

They are self-imposed, implicit and robust

Invent a subtraction problem whose solution is 8-3=5



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Intuitive conceptions of mathematics notions

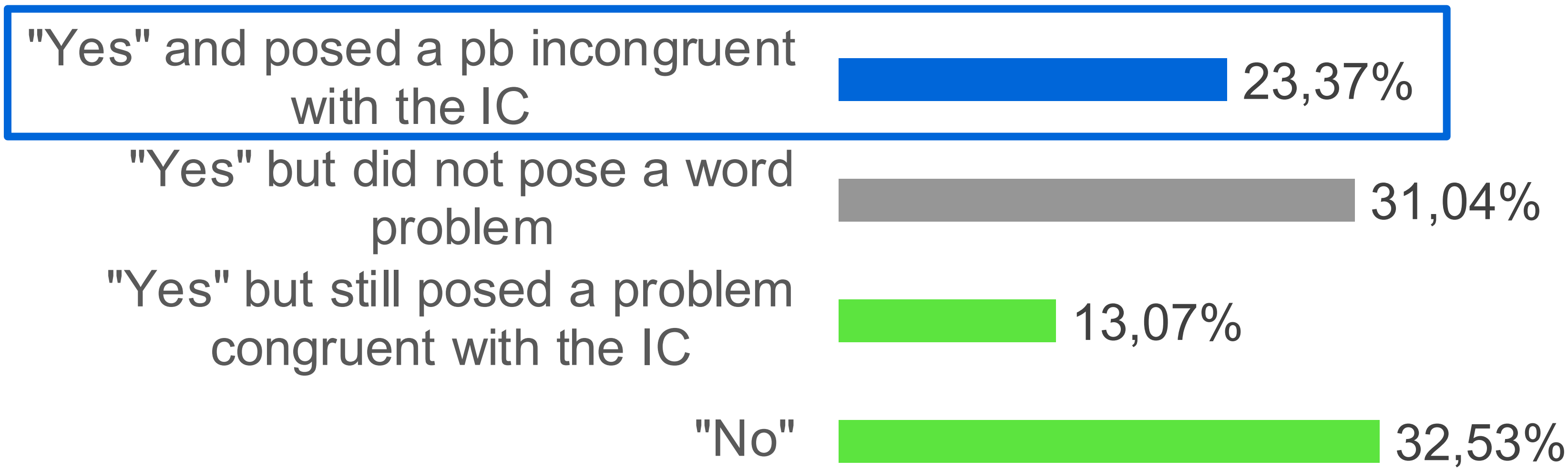
Subtraction:	Looking for what is left
Addition:	Looking for the total
Multiplication:	Iterated addition
Division:	Looking for the size of each part
Fractions :	Bipartite structure
Decimals :	Two numbers with a separator between them

The other cases are mathematically relevant but outside the scope of the intuitive conception: they need to be taught

And outside the scope of an intuitive conception?

Invent a subtraction problem whose solution is 8-3=5 and in which you don't lose anything, you only win

Paul has 3 marbles. He wins some during recess and now he has 8. How many marbles did he win?



Gvozdic, K., Naud, S., & Sander, E. (2022). How robust are intuitive conceptions? Insights from production tasks regarding arithmetic operations. https://doi.org/10.31234/osf.io/fny2m

And outside the scope of an intuitive conception?

J'ai 1 500 cartes.
J'ai 300 cartes de plus que de boites.
Combien ai-je de boites ?

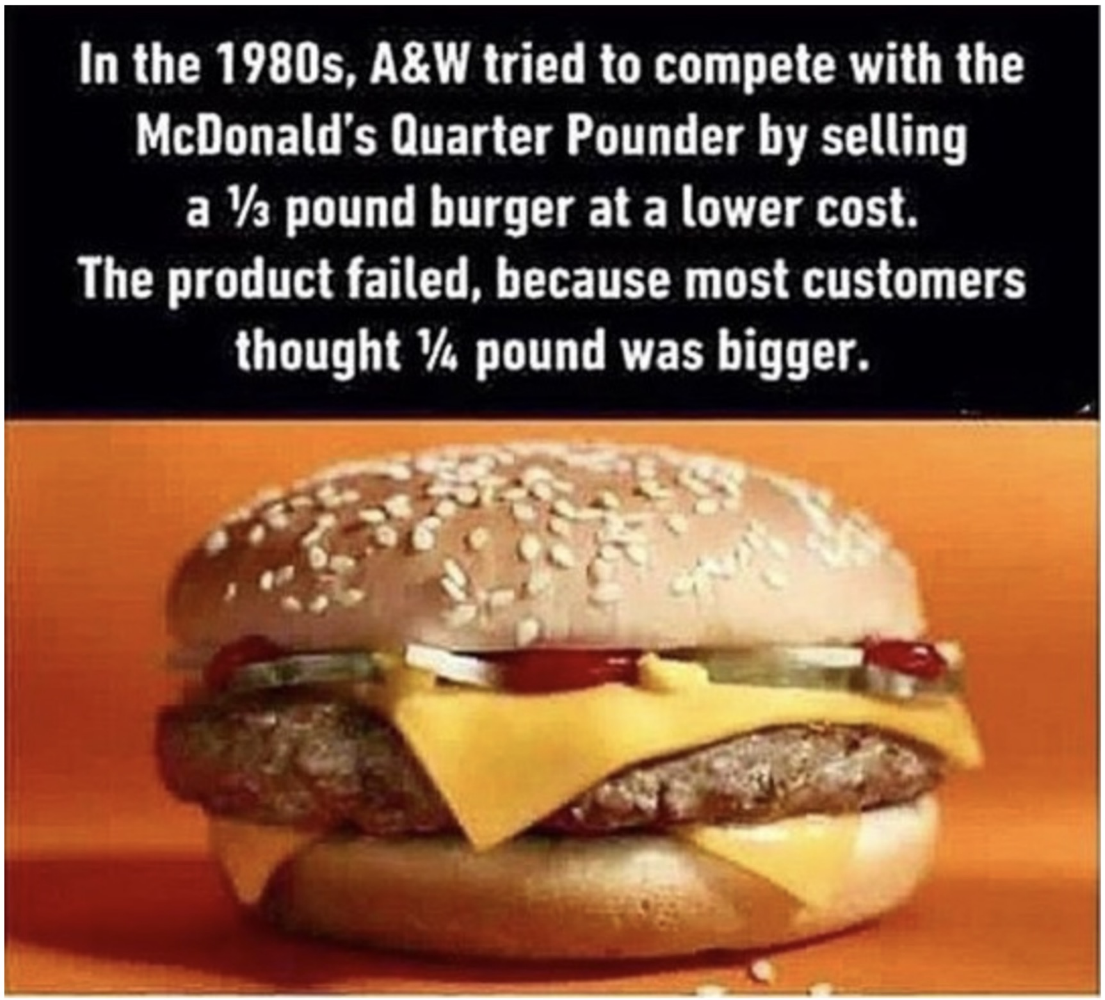
Cocher la bonne réponse :

	6e	4e
○ 1 800	22,4%	22,8%
○ 450 000	6,8,%	3,8%
○ 1 200	47,6%	48,9%
○ 5	18,1%	21,7%
No answer	5,1%	2,7%

644 6th grade students
548 8th grade students

DEPP, 2022

And outside the scope of an intuitive conception?



Educational entailments

A given situation is encoded by relying on features that are neither the most superficial aspects neither the more abstract one, but the ones perceived as the structural deep features by an individual.

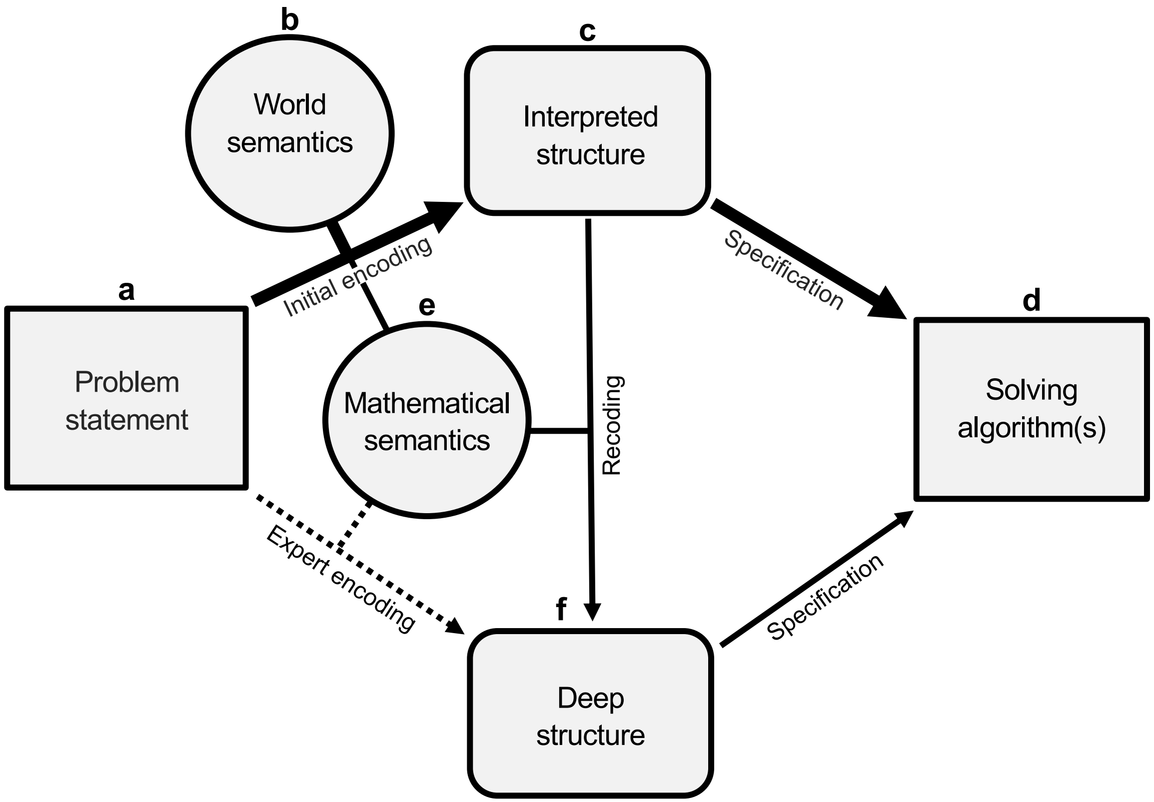
In case this initial encoding is not efficient and needs to be overcome, a semantic recoding process makes it possible for a student to perceive the situation in a way that is more in line with the mathematical knowledge that is targeted.

The educational entailments are important because it provides a way to foster learning at school.

Hypothesis: Recoding is key to mathematics success: it makes it possible to overcome the limitations of intuitive conceptions and of informal strategies

Hofstadter, D., & Sander, E. (2013). *Surfaces and Essences: Analogy as the fuel and fire of thinking.* New York, Basic Books.

The SECO Model



Gros, H., Thibaut, J.-P., & Sander, E. (2020), Semantic Congruence in Arithmetic: A New Conceptual Model for Word Problem Solving, *Educational Psychologist*.

Arithmetic Comprehension in Elementary school intervention

→Collaboration between mathematics education & cognitive psychology researchers and teachers

→Word problem solving domain:

Relate mathematical knowledge to real-world problems

Analyze on word problems through part-whole relations

Learn to use the most optimal solving strategies

Fischer, J.-P., Sander, E., Sensevy, G., Vilette, B. & Richard, J.-F. (2018). Can young students understand the mathematical concept of equality? A whole-year arithmetic teaching experiment in second grade. *European Journal a of Psychology of Education, 34*(2), 439-456.

Problem solving in ACE

- Combine problems → Introduction to word problem solving
- Compare problems → Finding the difference
- Combine problems → Semantic recoding
- Change problems → Practicing adaptive expertise

Influence of a semantic recoding activity in ACE

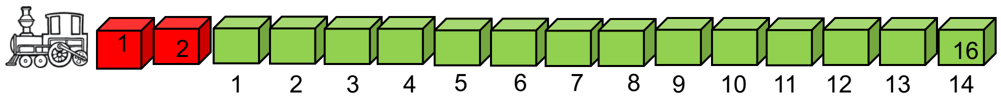
Influence of a semantic recoding activity at school devoted to help students perceive hidden analogies between problems so that they become opportunistic word problem solvers.

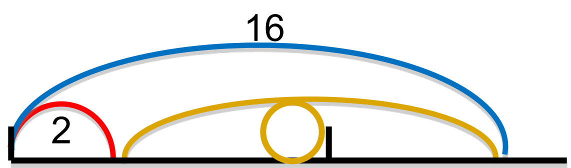
If a student succeeds in achieving a semantic recoding, they will be able to use the more efficient strategy even if it is not the one which is primed by the initial encoding. This will help them acquire a more general concept of the arithmetic operation and use formal arithmetic knowledge.

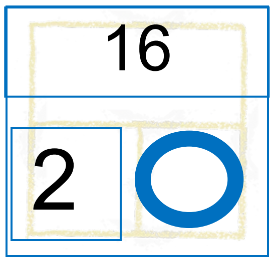
Gvozdic, K., & Sander, E. (2020). Learning to be an opportunistic word problem solver: Going beyond informal solving strategies. *ZDM Mathematics Education, 52(1*), 111-123

Semantic recoding in ACE

Zoey made a toy train.
Her train has 2 red wagons and some green wagons.
Altogether, her train has 16 wagons.
How many green wagons does her train have?







2+_=16

Semantic recoding in ACE

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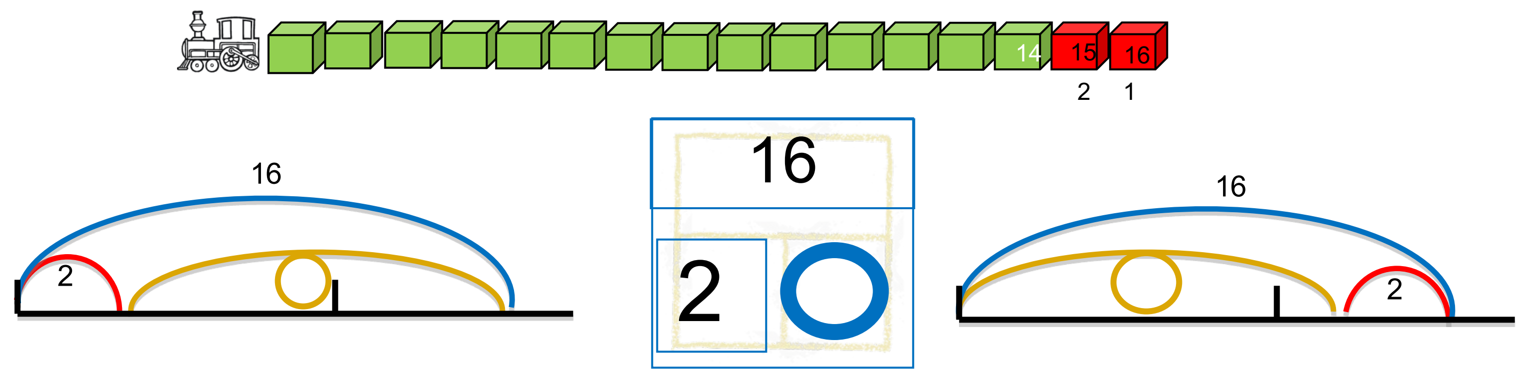


2+_=16

16-2=_

Zoey's train has 16 green wagons.

Experimental Design

Control Group
- 103 students
- CM — 1st Grade
- BAU

Experimental Group
- 105 students
- CM — 1st Grade
- Intervention emphasizing recoding

Test (no pretest but control tasks)

Method

Materials: 12 arithmetic word problems (6 semantic categories)

- 6 low-cost mental simulation problems

= problems that are easy to solve based on the initial encoding of the situation
→ informal strategy is efficient

- 6 high-cost mental simulation problems

= problems that are difficult to solve based on the initial encoding of the situation → informal strategy is inefficient

→ benefits of a recoded representation observed through formal strategy use

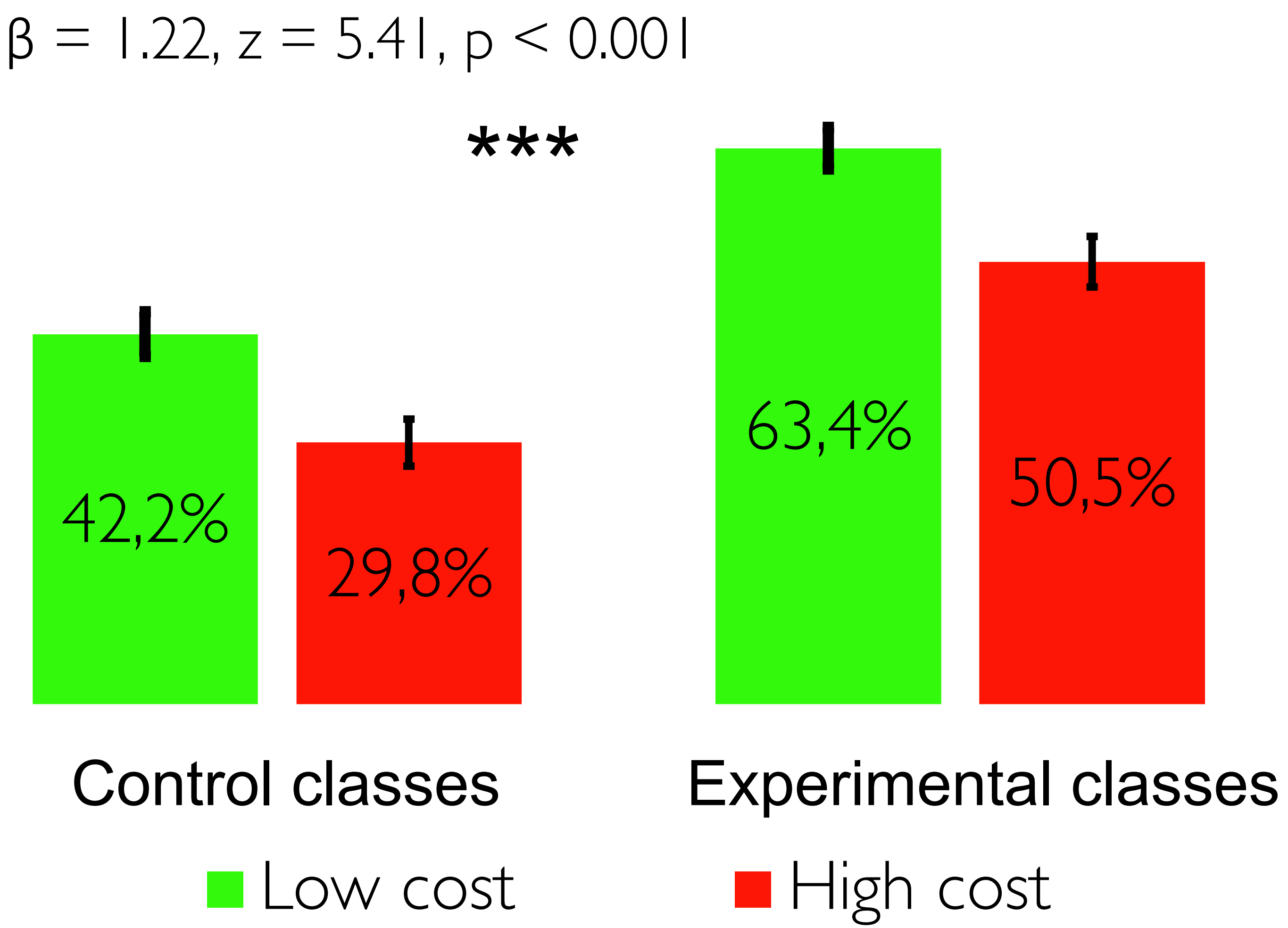
Measure: correct responses & solving strategies + control tasks

Participants: 1st grade students :

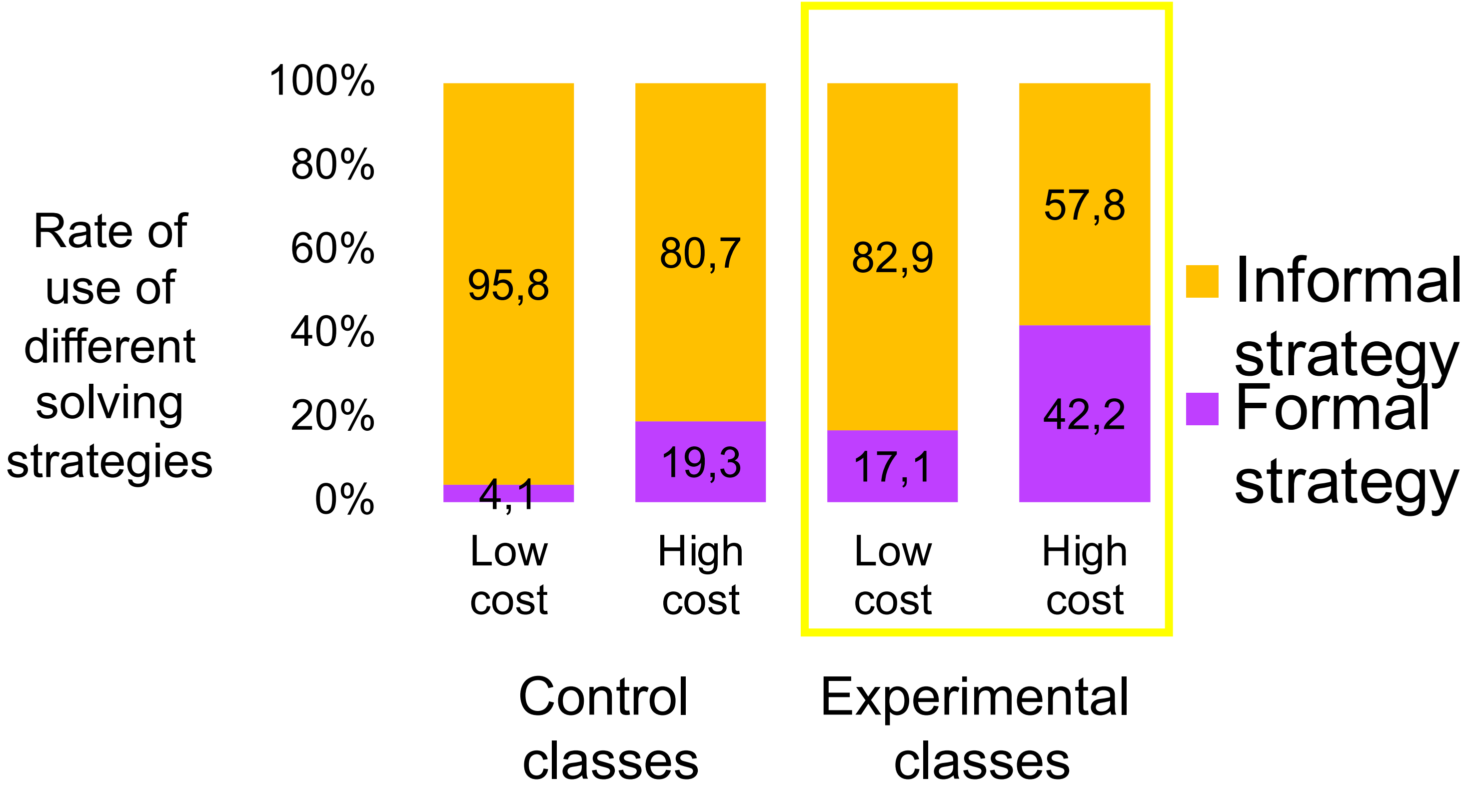
- 5 business as usual classes (control) (N = 103, 7.05 y/o)
- 5 ACE classes (experimental) (N = 105, 7.03 y/o)

Predictions: better performance + greater use of formal strategies among experimental classes (ACE)

Problem solving performance



Problem solving strategies



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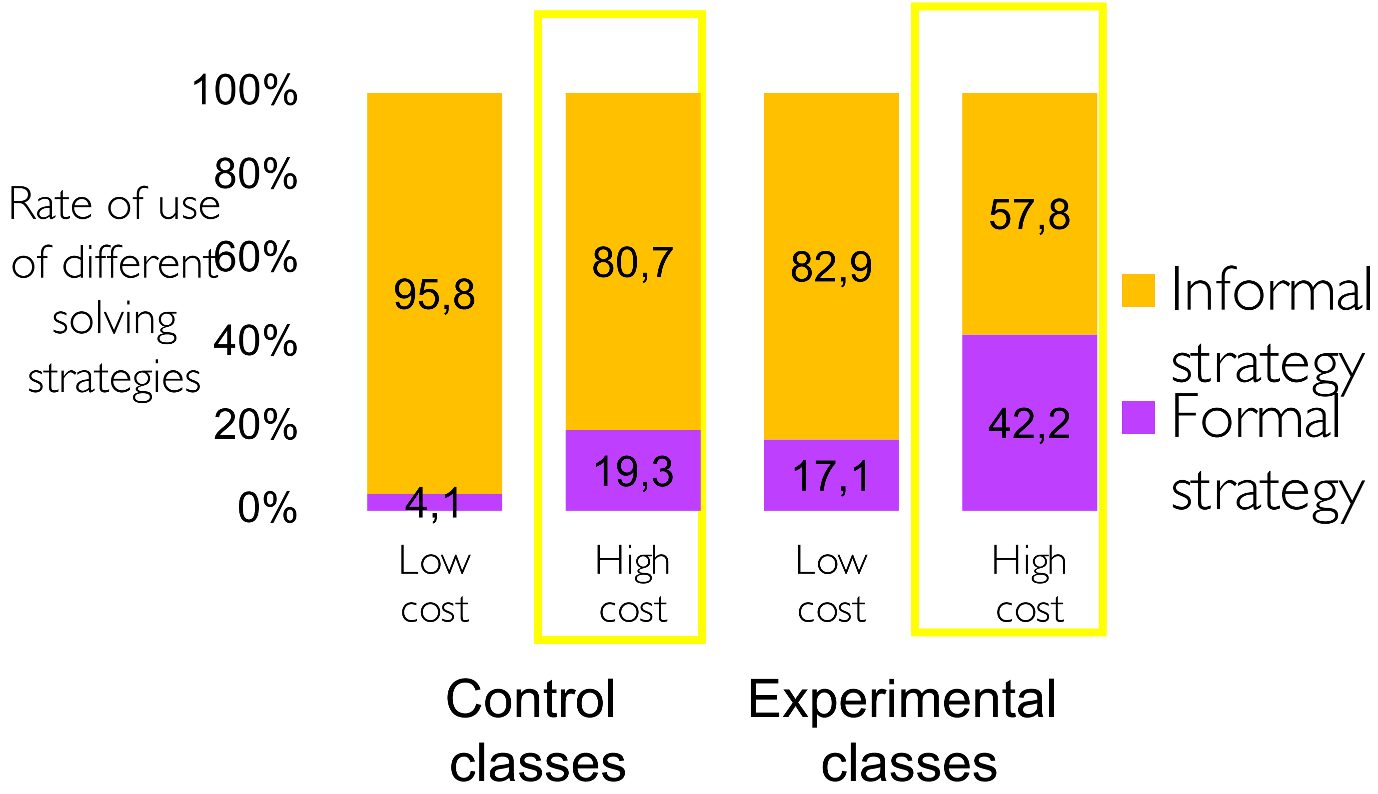
Problem solving strategies



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Main findings overview

- Higher overall performance
  - Experimental > Control
- Higher performance on problems for which a recoded representation is most beneficial
  - Experimental > Control on high cost problems

- Higher overall use of formal solving strategies
  - Experimental > Control
- Higher use of formal solving strategies on problems for which a recoded representation is most beneficial
  - Experimental > Control on high cost problems

Discussion

- Low-cost mental simulation problems provided opportunities to use solving strategies in various contexts and enrich procedural knowledge, while high-cost mental simulation problems provided opportunities to invent and search for alternative strategies and develop conceptual knowledge

- ACE students from the experimental classes succeeded better on problems where the informal strategy of first resort must be countered, and they had a greater range of problem-solving strategies. Most importantly, formal strategies were used adaptively – on problems where their application is most advantageous

- ➔ Semantic recoding thus seems to be a promising approach to conceptual change

RAIFLEX Intervention

Students are often misled by their intuitive conceptions regarding mathematical components of proportional reasoning

Limitations can be mitigated by working on alternative ways of categorizing situations, that lead to a more adaquate encoding than the initial one

Multiple categorization triggers flexibility, which enables reinterpreting a problem statement and adopting a more relevant point of view

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RAIFLEX Intervention

Proportional reasoning however also relies on several preconceptions that are constraining and impose limits for reaching an expertise and flexibility in solving proportional problems.

Multiplication as Repeated Addition

Division as Sharing

Fraction as a Bipartite Structure

The Illusion of Linearity

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Nina a 7 caisses et des pommes.
Elle a 21 fois moins de caisses que de pommes.
Combien a-t-elle de pommes ?

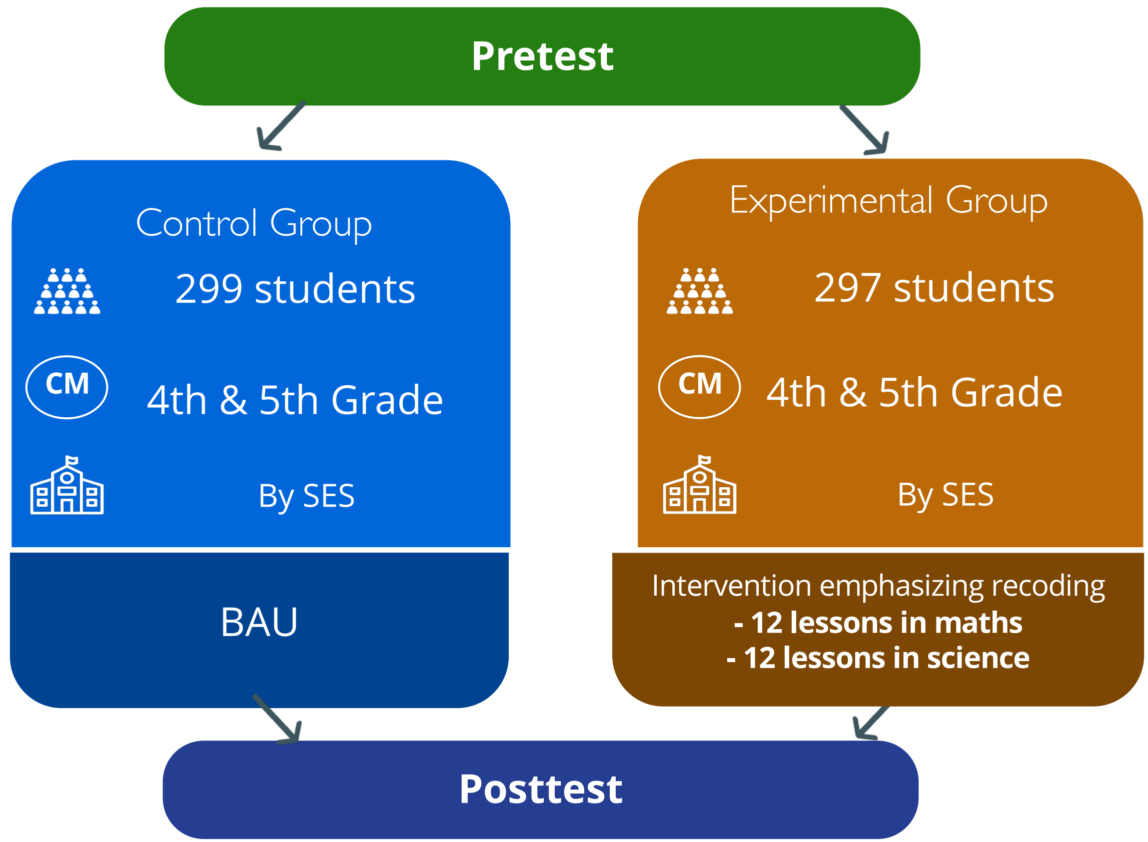
Cocher la bonne réponse :

	6e	4e
○ 3	**17%**	**24,0%**
○ 14	**19,3%**	**14,7%**
○ 147	**34,2%**	**39,5%**
○ 28	**21,9%**	**16,3%**
No answer	**7,6%**	**5,4%**

DEPP, 2022

688 6th grade students
570 8th grade students

Experimental Design



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The intervention program

Modalities of learning sessions

Math 12 lessons

Science 12 lessons

1h

BO LE BULLETIN OFFICIEL DE L'ÉDUCATION NATIONALE

- Sessions codesigned with a teacher-trainer
- 2 hours of teacher training on intuitive conceptions and multiple categorization
- Support for teachers by the teacher trainer
- 12 math lessons focusing on flexibly envisioning a situation involving proportional reasoning from different points of view

The intervention program

3 times more and 3 times less

Problem 1 :

Lisa has 7 red cubes and 24 red flowers.
Leo has 21 blue cubes and 6 blue flowers.

1) Who has the most cubes? How many times more?

Point of view ________________________

______ has _____times _______cubes than______.

2) Who has the least cubes? How many times less?

Point of view ________________________

______ has _____times _______cubes than______.

3) Who has the least flowers? How many times less?

Point of view ________________________

______ has _____times _______flowers than______.

4) Who has the most cubes? How many times more?

Point of view ________________________

______ has _____times _______flowers than_____.

The intervention program

Exemplary Summary of a Teacher's Guide Sheet

Intuitive conceptions: Times more as repeated addition, times less as a repeated subtraction

Conception to build: times more/times less as a ratio

Learning challenges:
Move from the point of view of "repeated addition" (3 + 3 + 3 + 3) to the notion of product (4 x 3).
Adopt the « times more » and « times less" points of view
Understand that multiplication and division are about finding a ratio

Points of view to be adopted	Key sentences to be said
« times more » point of view « times less" point of view	- Saying that X has 3 times more than Y is the same as saying that Y has 3 times less than X. - you must ask yourselves "it is X times more/less in relation to whom/what?"

The intervention program

Pourquoi cette étape ?
On donne le rapport « 4 fois moins », et il faut retrouver la quantité.
On confronte « fois moins » et « de moins ».

Problème de référence :
Yanis a 28 cubes. Elsa en a 4 fois moins. Et Jérémie a 4 cubes de moins que Yanis. Qui en a le plus ? Combien Yanis a-t-il de cubes ? Combien Elsa a-t-elle de cubes ?

Prénom :

Enoncé 2 : Yanis a 28 cubes. Elsa en a 4 fois moins. Et Jérémie a 4 cubes de mois que Yanis.

1) Qui en a le plus ? Yanis.

2) Combien Jérémie a-t-il de cubes ?

28 - 4 = 24 cubes Jérémie a 24 cubes.

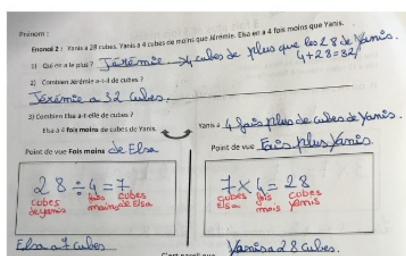
3) Combien Elsa a-t-elle de cubes ?

Elsa a 4 fois moins de cubes de Yanis. — Yanis a 4 fois plus de cubes qu'Elsa.

Point de vue d'Elsa / fois moins	Point de vue de Yanis / fois plus
? = 28 : 4 (7) = 28 : 4	28 = 4 x ? 28 = 4 x (7)
Elsa a 7 cubes.	Elsa a 7 cubes.

C'est pareil que

Dans les écritures mathématiques, on précise bien les unités.



On réinvestit la notion de « de moins » vu en séance 1.

On fait reformuler la phrase en changeant de point de vue : on prend celui de Yanis / fois plus, car ca va être plus facile pour calculer.

Yanis a 4 fois plus de cubes qu'Elsa : on part du nombre de cubes de Yanis : 28 = 4 x ?

Elsa a 4 fois moins de cubes que Yanis : On part du nombre de cubes d'Elsa, c'est ce qu'on recherche, on met ? = 4 fois moins que 28 = 28 : 4.

The intervention program

Principles design of lessons

Principle 1:
Overcoming intuitive conception

Principle 2:
Favor the adoption of multiple points of views on a situation

Principle 3:
Explain and make explicit the different points of views to the students

Principle 4:
Diversify learning contexts - transfer of the same reasoning

Principe 1 : Overcoming intuitive conceptions

	Intuitives Conceptions	Targeted Conceptions
1 – From additive to multiplicative langage	En plus = Fois plus	Fois plus comme rapport
2 – Multiplicative langage	Fois plus / Fois moins comme addition /soustraction réitérée	Fois plus / Fois moins comme la recherche du rapport
3 – Exchange problems	Multiplier pour avoir plus et Diviser pour avoir moins	Multiplier et diviser pour trouver un rapport entre deux grandeurs
4 – Parts or Whole Point of views	Chaque = Tous	Compréhension des quantifieurs. Distinguer struct. add. et mult.
5 – Fraction from what ?	Fraction bipartite	Fraction comme rapport
6 – Partitive and Quotitive DIvision	Diviser pour partager	Diviser pour mesurer
7 – Equivalence between division and multiplication by a fraction	Fraction bipartite	Fraction comme rapport
8 – Proportion (1)	Proportion comme écart à conserver	Proportion comme rapport à conserver
9 – Proportion (2)		
10 – Proportionnal reasoning – 3 strategies	Retour à l'unité	Recherche du rapport
11 – Proportionnal reasoning – 4 stratégies		

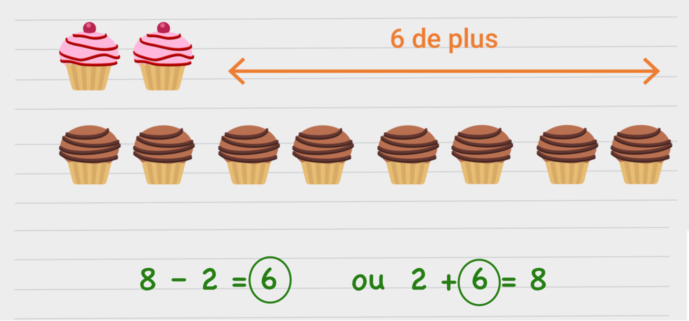
Scheibling-Sève, C., Gvozdic, K, Pasquinelli, E., & Sander, E. (2022) Enhancing cognitive flexibility through a training based on multiple categorization: developing proportional reasoning in primary school. *Journal of Numerical Cognition, 8(3)*, 443-472.

Math lesson

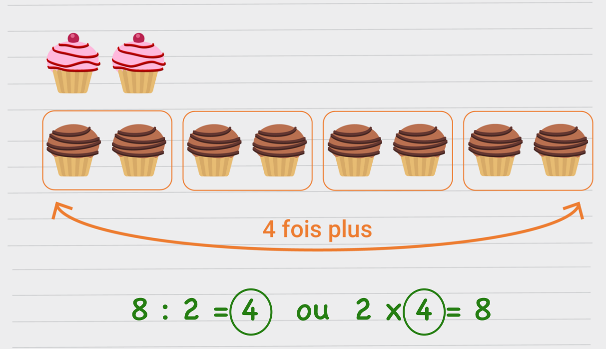
I have 2 cherry muffins. I have 8 chocolate muffins.

Do I have more cherry or chocolate muffins?
How many more? How many times more?

Point of view : More



Point of view: Times more

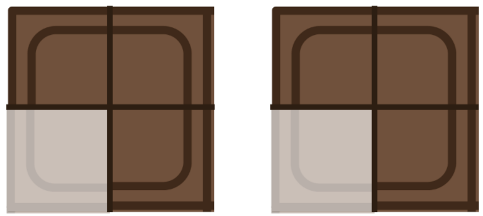


Math lesson

J'ai mangé **un quart de 2 carreaux** de chocolats.

Propose plusieurs façons, écris en Math et en Français

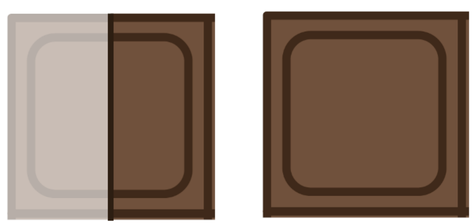
Point of view: Parts



En math : $\frac{1}{4} + \frac{1}{4} = \frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$

En français : *Un quart de chaque carreau*

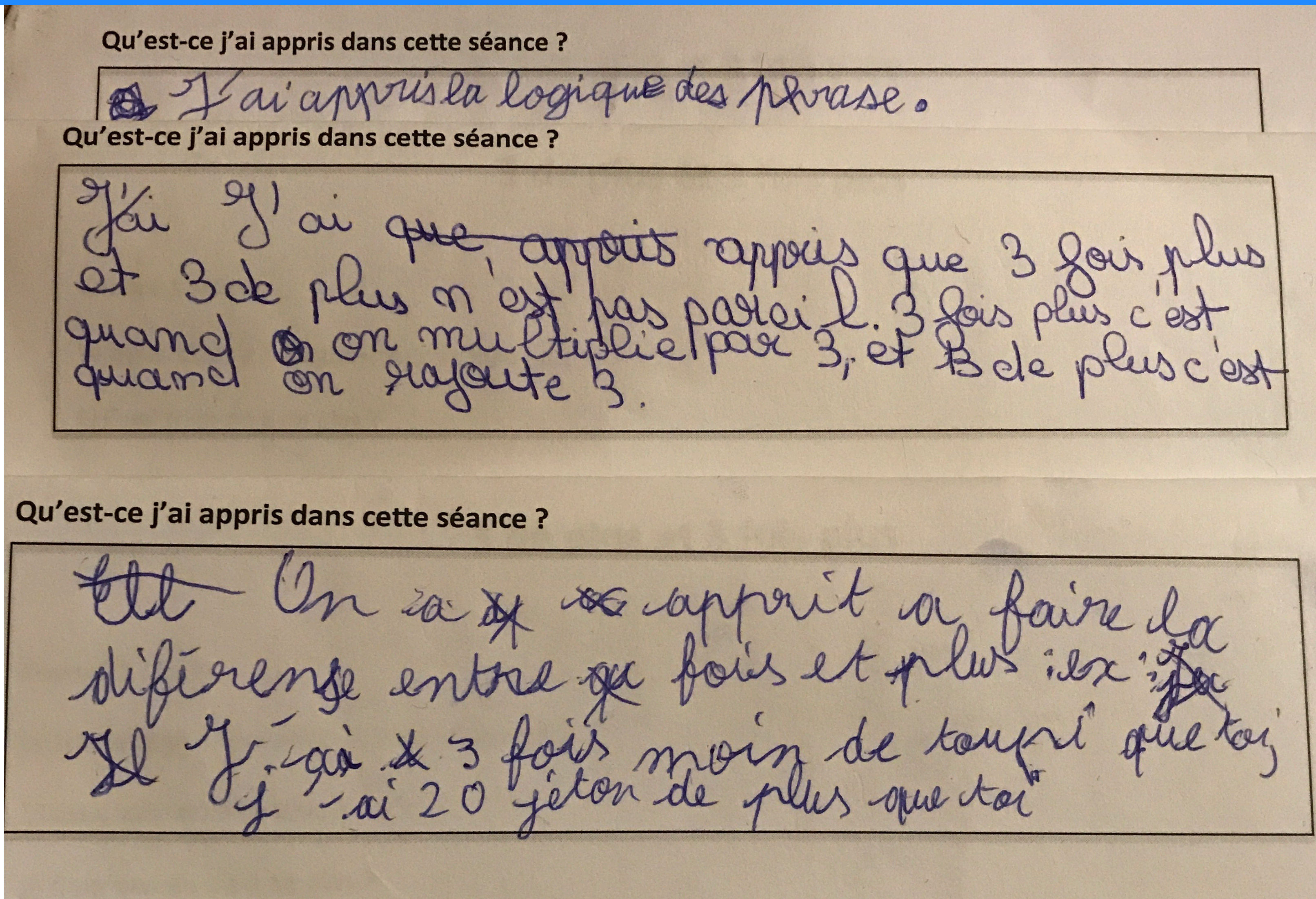
Point of view: Whole



En math : $\frac{2}{4} \times 1 = \frac{2}{4} = \frac{1}{2}$

En français : *Deux quarts de un carreau*

Principe 3 : Explain and make explicit the different points of view to the students



Qu'est-ce j'ai appris dans cette séance ?

J'ai appris la logique des phrase.

Qu'est-ce j'ai appris dans cette séance ?

J'ai ~~que apprits~~ appris que 3 fois plus et 3 de plus n'est pas pareil. 3 fois plus c'est quand on multiplie par 3, et 3 de plus c'est quand on rajoute 3.

Qu'est-ce j'ai appris dans cette séance ?

On a apprit a faire la diférense entre fois et plus ex : J'ai 3 fois moin de tourni que toi ; j'ai 20 jéton de plus que toi

Principle 4 : Diversify learning contexts - transfer of the same reasoning

Problème de 4e proportionnelle | Retour à l'unité | Rapport

À la boulangerie, Juliette achète 9 **croissants** et paye 12€.

J'achète 3 **croissants**. Je vais donc payer ....

Point of view: Unit

9 croissants = 12 €

1 croissant = $\frac{12}{9}$

3 croissants = $\frac{12}{9} \times 3 = \frac{36}{9} = 4$ €

Point of view: Times less

9 croissants = 12 €

÷3 fois moins

3 croissants = 4 €

÷3 fois moins

Point of view: Proportion

9 croissants = 12 €

3 croissants = ?

3 croissants, c'est le tiers de 9 croissants.

Le tiers de 12€, c'est $12 \times \frac{1}{3} = 12 \div 3 = 4$€

Point of view: TImes more

9 croissants = 12 €

x3 fois plus

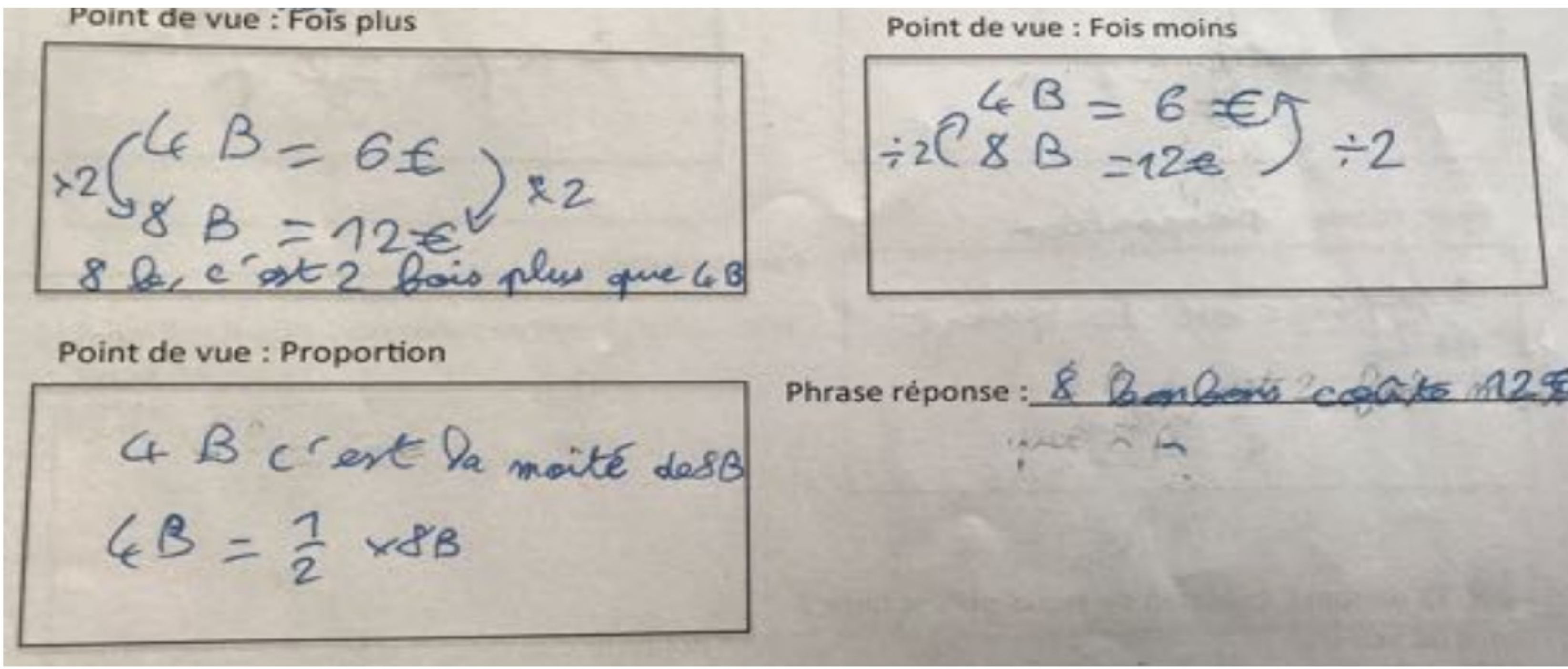
3 croissants = 4 €

x3 fois plus

Problème : J'ai 4 bonbons pour 6 €. Combien coûtent 8 bonbons ?

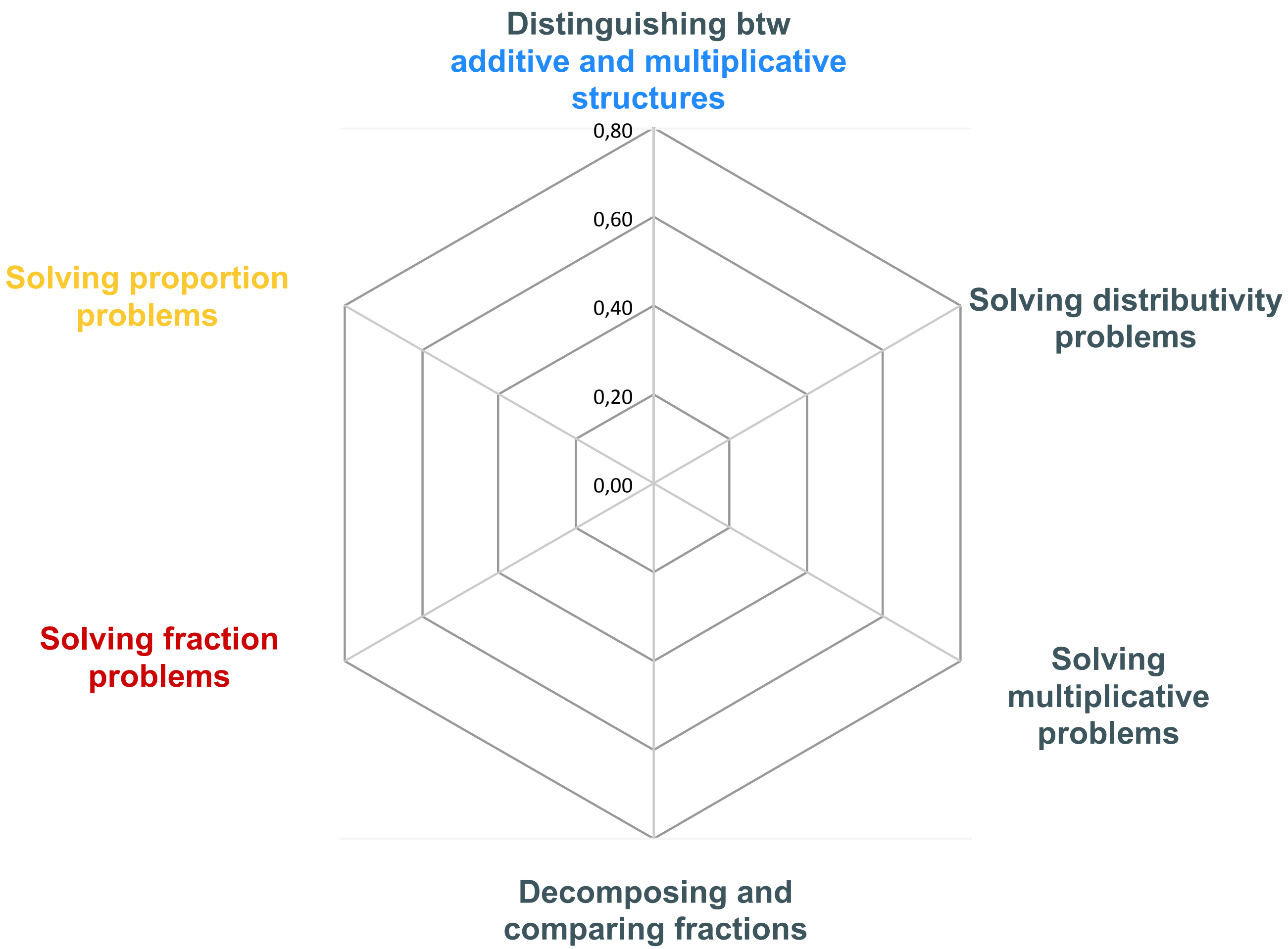
On commence par faire résoudre le problème sur l'ardoise. Tous les élèves trouvent ainsi la réponse. Puis on explique que maintenant, on va s'attacher à retrouver ce résultat de 3 façons différentes.

Dans le premier cadre, on écrit : « 4b = 6€ ». Les élèves doivent poursuivre.



Pretests
Posttests

35 problems
- TIMMS
- French Ministry
- Research
- Created for the study

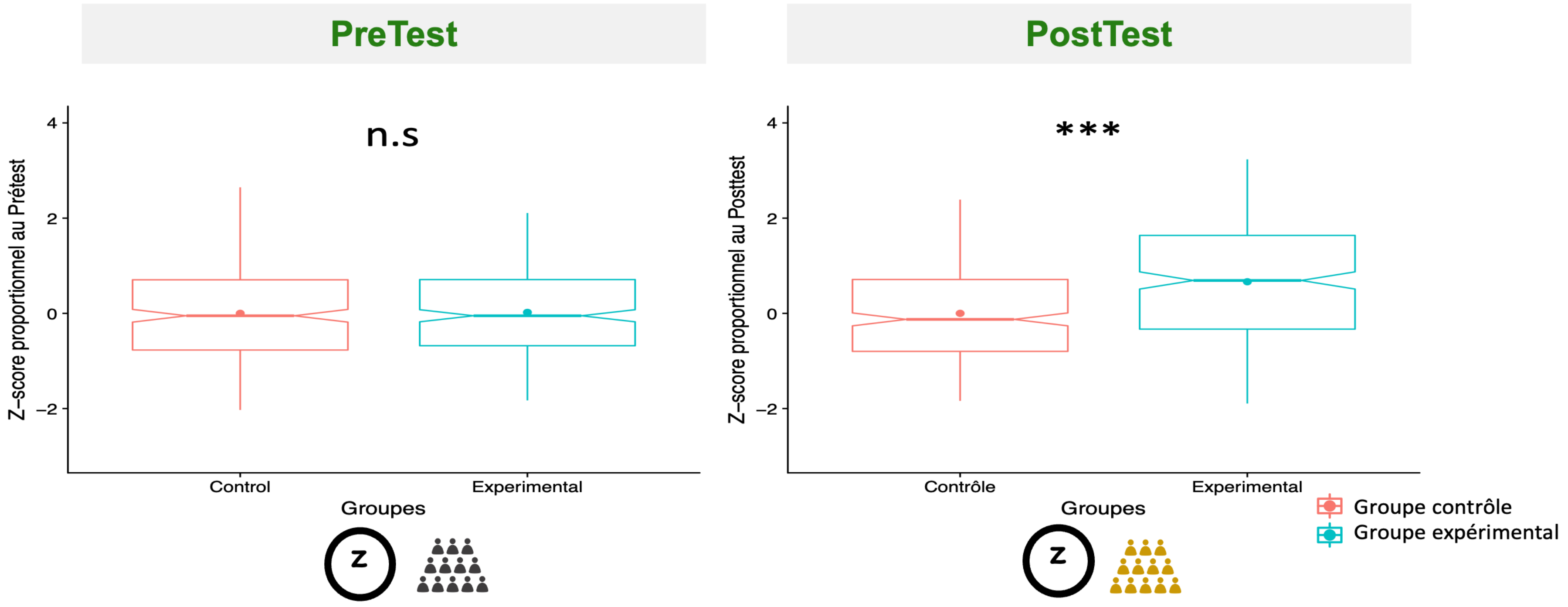


Hypotheses

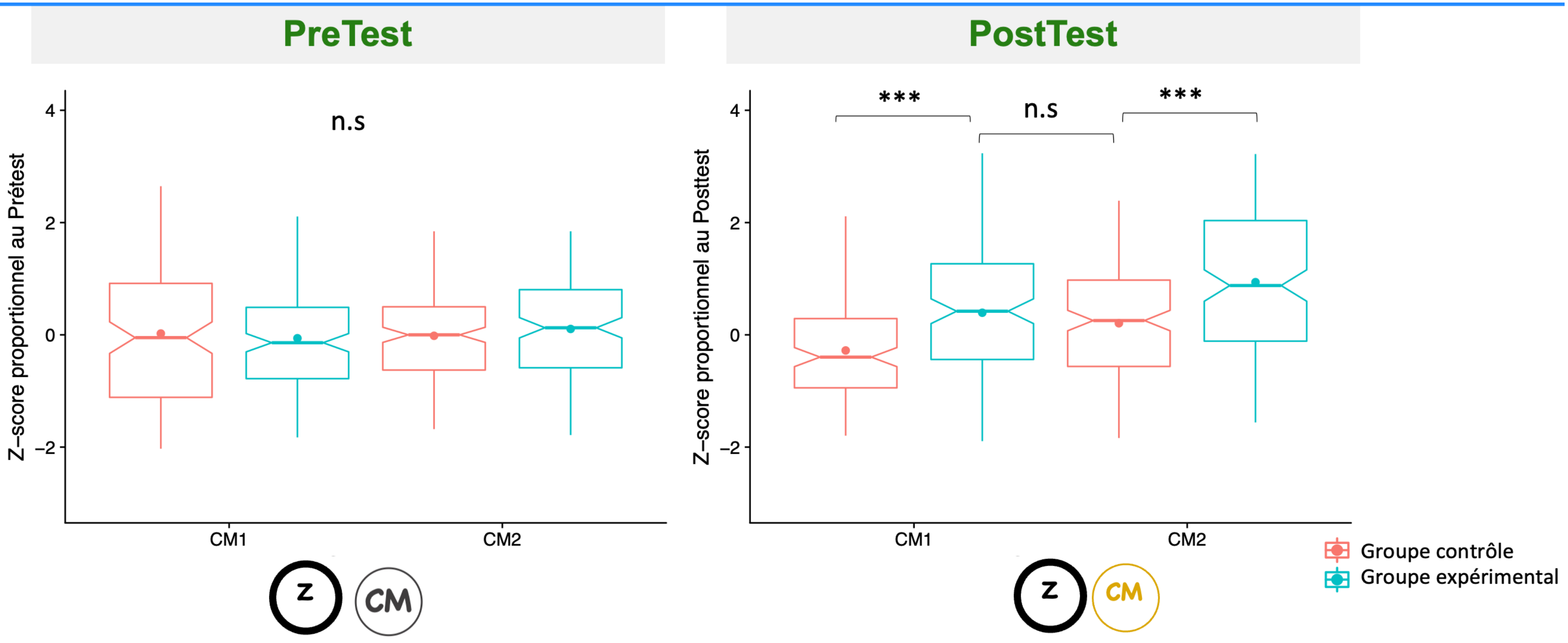
On the pre-test, there should be no difference in performance between the experimental and control groups.

At post-test, the experimental groups should perform better than the control groups.

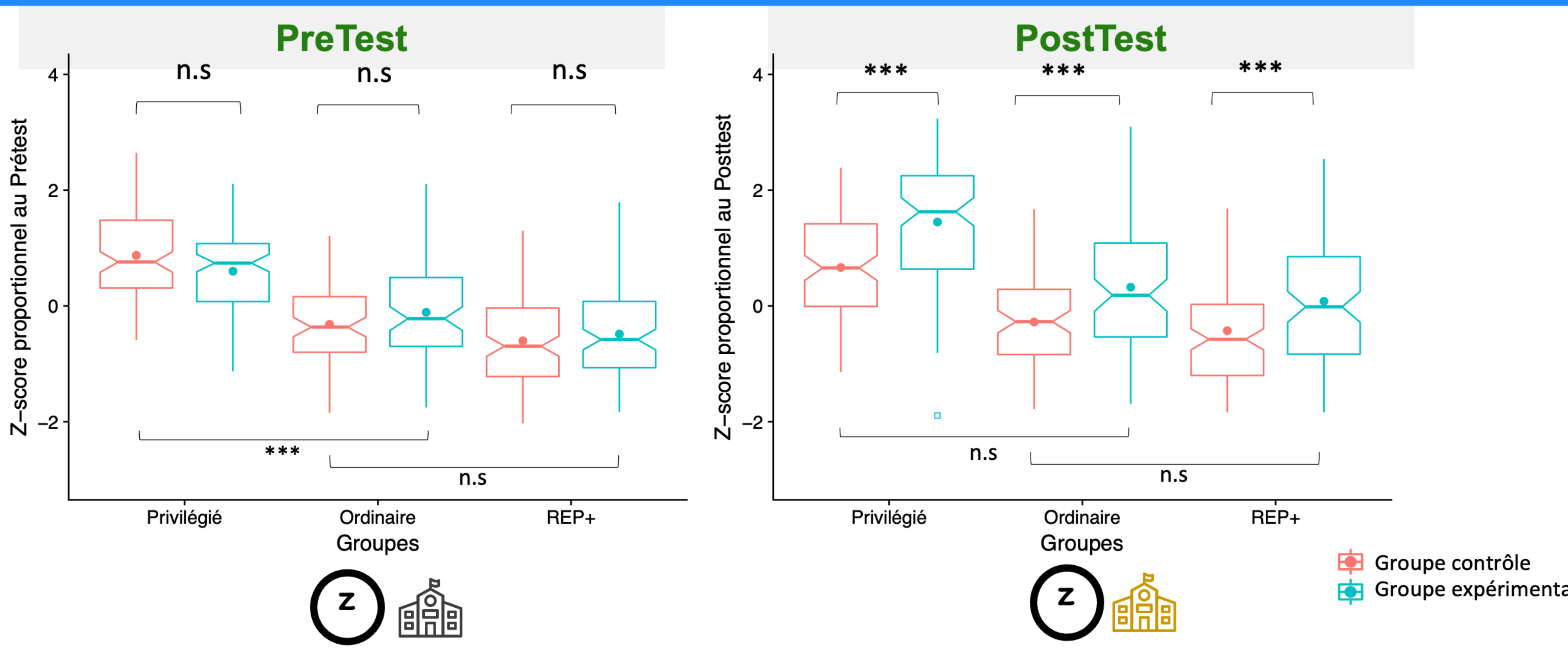
Results at Global Level



Results by Grades



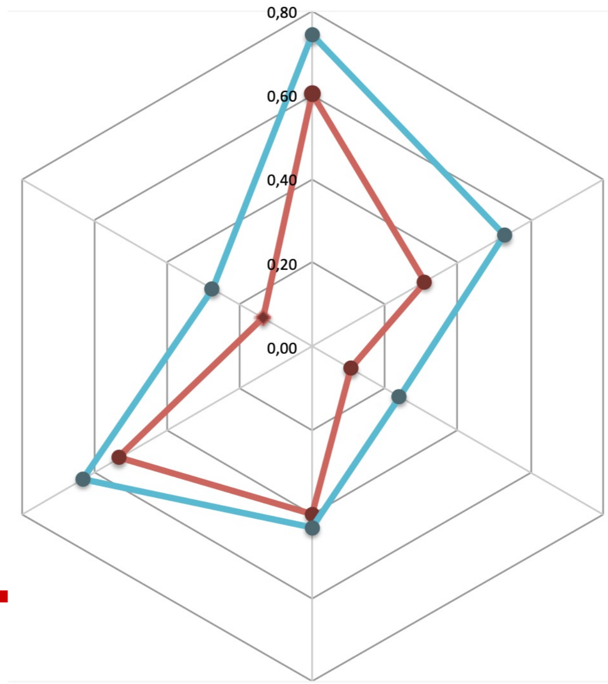
Results by SES



Results by Sub-Scores

Distinguishing btw additive and multiplicative structures

CM1 = CM2 CM



Résoudre des problèmes de proportion

CM1 = CM2 CM

Résoudre des problèmes de distributivité

CM1 > CM2 CM

Résoudre des problèmes fractionnaires

CM1 = CM2 CM

Résoudre des problèmes multiplicatifs

CM1 > CM2 CM

Décomposer et comparer des fractions

CM1 = CM2 CM

Groupe expérimental
Groupe contrôle

Main findings overview

- On the pre-test, there was no difference in performance between the experimental and control groups

- Experimental group > Control group

→ At global level
→ By grades
→ By SES
→ By Sub-Scores (all but one differences were significant)

Discussion

- When a problem can be solved with several strategies, it can be particularly beneficial to work on the conceptual knowledge to which each strategy is attached.

➢ Flexibility cannot simply refer to the smooth transition between several strategies, but that achieving flexibility mobilizes the complex relations between conceptual and procedural knowledge.

➢ Intervention and its assessment conducted in this study highlight the usefulness of overcoming some on the limitations of an initial representation constrained by intuitive conceptions and informal strategies, and also provide insight into the benefits for fostering students' flexibility in strategy use.